

Using Math to Talk About Cells Answers: Surface-to-Volume Ratios in Biology

This lesson is based on a lesson developed by Dr. Stephen G. Saupe; College of St. Benedict/ St. John's University; Biology Department; Collegeville, MN 56321; ssaupe@csbsju.edu. I found it at <http://employees.csbsju.edu/ssaupe/biol327/Lab/sv-ratio.htm>

In order for cells to survive, they must constantly exchange ions, gases, nutrients, and wastes with their environment. These exchanges take place at the cell's surface and limit cell growth. These exercises are designed to introduce the concept of surface area, volume, and surface-to-volume ratios (S/V) and their importance in biology.

Surface area is the summation of the areas of the exposed sides of an object and volume is a measure of how much space an object occupies. The surface to volume ratio, or S/V ratio, refers to the amount of surface a structure has relative to its size. To calculate the S/V ratio, simply divide the surface area by the volume. We will examine the effect of size, shape, flattening an object, and elongating an object on surface-to-volume ratios.

To perform this function efficiently, there must be an adequate ratio between the cell's volume and its surface area.

I. INFLUENCE OF SIZE ON S/V RATIOS.

The purpose of this exercise is to see how the S/V changes as an object gets larger. We will use a cube to serve as a model cell (or organism). Cubes are especially nice because surface area (length x width x number of sides) and volume (length x width x height) calculations are easy to perform. To calculate the surface-to-volume ratio, divide the surface area by the volume. Complete the table below for a series of cubes of varying size:

Length of side (mm)	Surface Area (mm ²)	Volume (mm ³)	Surface/Volume ratio
1	6	1	6
2	24	8	3
3	54	27	2
4	96	64	1.5
5	150	125	1.2
6	216	216	1.0
7	294	343	0.86
8	384	512	0.75
9	486	729	0.67
10	600	1000	0.6

Questions and Analysis:

1. Which cube has the greatest surface area? Volume? S/V ratio?
2. What happens to the surface area as the cubes get larger?
3. What happens to the volume as the cubes get larger?
4. What happens to the S/V ratio as the cubes get larger?
5. Proportionately, which grows faster: surface area or volume? Explain.

6. Which cube has the most surface area in proportion to its volume?
7. If you cut a cube in half, how does the volume, surface area, and S/V ratio of one of the resultant halves compare to the original?
8. Plot, all on one graph, the following: S/V ratio vs. cube size (length in mm); volume vs. cube size (length in mm); and surface area vs. cube size (length in mm).

II. S/V RATIOS IN FLATTENED OBJECTS:

In this exercise we will explore how flattening an object impacts the surface to volume ratio. Consider a box that is 8 x 8 x 8 mm on a side. Then imagine that we can flatten the box making it thinner and thinner while maintaining the original volume. What will happen to the surface area and S/V ratio as the box is flattened? Complete the table below.

Box No.	Height (mm)	Length (mm)	Width (mm)	Surface area (mm ²)	Volume (mm ³)	S/V Ratio
1	8	8	8	384	512	0.75
2	4	16	8	448	512	0.88
3	2	16	16	640	512	1.3
4	1	32	16	1120	512	2.2
5	0.5	32	32	2112	512	4.1

Questions/ Analysis:

1. Explain why leaves are thin and flat.
2. Why do elephants have large, flat ears?
3. Explain why desert plants generally have smaller leaves.

III. S/V RATIOS IN ELONGATED OBJECTS:

In this exercise we will explore how elongating an object impacts the surface to volume ratio. Consider a box that is 8 x 8 x 8 mm on a side. Then, imagine that we pull on the ends to make it longer and longer while maintaining the original volume. What will happen to the surface area, and S/V ratio as the box is flattened? Complete the table below.

Box No.	Length (mm)	Height (mm)	Width (mm)	Surface area (mm ²)	Volume (mm ³)	S/V ratio
1	8	8	8	384	512	0.75
2	16	4	8	448	512	0.88
3	32	4	4	544	512	1.1
4	64	2	4	784	512	1.5
5	128	2	2	1032	512	2.0

Questions/ Analysis:

1. Explain the shape of blood vessels.
2. Explain why roots have "hairs".

IV. SHAPE AND S/V RATIOS:

In this exercise we will explore the impact of shape on surface to volume ratios. The three shapes given below have approximately the same volume. For each, calculate the volume, surface area, and S/V ratio and complete the table. The last column in the table, “Volume of environment within 1.0 mm of the object” is particularly important. Since the materials that an organism exchanges with its environment come from its immediate surroundings, the greater this volume, the more material that can be exchanged.

Shape	Dimensions (mm)	Volume (mm ³)	Surface area (mm ²)	S/V ratio	Volume (mm ³) of the environment within 1.0 mm of object
Sphere	1.2 diameter	.90	4.5	5.0	16.3
Cube	1 x 1 x 1	1.0	6.0	6.0	26
Filament	0.1 x 0.1 x 100	1.0	40	40	449

Note: volume of a sphere = $\frac{4}{3}(\pi)r^3 = 4.189r^3$. Surface area of sphere = $4(\pi)r^2 = 12.57r^2$
A filament is a box. To calculate surface area of a box determine the surface area of each face (l x w) and then add them. The volume of a box = l x w x h.

Questions & Analysis:

1. Make a sketch, to scale, of the three objects.
2. Which shape has the greatest surface area? Volume? S/V ratio?
3. If you had to select a package with the greatest volume and smallest surface area, what shape would it be?
4. Explain why cells divide as they get larger.
5. Explain why the shape of animals is basically “spherical”, whereas plants and fungi are “filamentous”.
6. Given that one way a plant grows is cell elongation. How does the plant’s vacuole help accomplish this task and why does this help solve the S/V ratio conundrum?